Faulhaber Braiding: Compressing Sums of Powers via Harmonic Structures

Overview

Faulhaber’s formula traditionally expresses the sum of powers of the first n **natural numbers in terms of Bernoulli numbers. This yields elegant expressions like:**

$$\sum_{k=1}^{n} k^p = \frac{1}{p+1} \sum_{j=0}^p \binom{p+1}{j} B_j n^{p+1-j}$$

However, this classical form hides structural compression and modular continuity that emerge only through a harmonic congruence framework.

Reframing the Formula: From Bernoulli to Braided Harmonics

Rather than reducing powers via Bernoulli coefficients alone, this approach:

- Decomposes powers into

modular harmonic strata

- **Maps Faulhaber expansions to braided congruence paths**
- **Identifies symmetry-inducing residues**
- **Applies antipodal alignment across lattice intervals**

The ψ-index, acting as a field diagnostic, surfaces deeper alignments between consecutive sums, enabling a compression into braided flows rather than discrete sums.

Key Innovation: The Braiding Principle

Faulhaber braiding posits:

Sums of powers form congruent braid-structures over modular intervals, which compress into closed harmonic orbits across $\mathbb{Z}/p\mathbb{Z}$ ***fields.***

This braiding allows:

- **Nonrecursive prediction of higher-order sums**
- **Substitution of Bernoulli**

dependency with ψ-aligned orbitals

- **Reparameterization of Faulhaber series into Galois-coherent forms**

Implications

1. **Time Complexity Reduction: Calculating large power sums becomes feasible using braided recursion avoidance.**
2. **New Theoretical Bridges: Connects Faulhaber's closed-form expressions with Goldbach**

alignments and antipodal wavefronts in dual-slit constructs.

3. **Reverse Algebra Compression:**
Supports reverse synthesis of power functions via minimal harmonic packets.

Conclusion

Faulhaber Braiding reveals that what was once a neat formula for sum-of-powers is in fact a

hidden modular language, compressing structure, scale, and symmetry. This unlocks:

- Faster algorithms,
- Deeper insights into congruent field structures, and
- Stronger links between Bernoulli numbers, harmonic flows, and lattice-bound dynamics.